Fuzzy Logic

Overview of Lecture

1) Difference between Fuzzy Logic and Boolean Logic
2) Fuzzy sets and its properties
3) Membership Function
4) Fuzzification and Defuzzification
5) Linguistic Hedges
6) Fuzzy Inference System
7) Fuzzy Control System
8) Application Areas

History of Fuzzy Logic

- 1965: First Paper "Fuzzy Logic" by Prof. Lotfi Zadeh, Faculty in Electrical Engineering, U.C. Berkeley, sets the foundation stone for the "Fuzzy Set Theory"
- 1970 Fuzzy Logic applied in control Engineering.
- 1975 Japan makes an entry
- 1980 Empirical Verification of Fuzzy Logic in Europe Broad Application of Fuzzy Logic in Japan
- 1990 Broad Application of Fuzzy Logic in Europe and Japan
- 1995 U.S increases interest and research in Fuzzy Logic.
- 2000 Fuzzy Logic becomes a Standard Technology and is widely applied in Business and Finance.

Introduction

1) FL is a branch of mathematics that allows a computer to model the real world the same way that people do.
2) FL provides a simple way to reason with vague, ambiguous, imprecise and noisy input or Knowledge.
3) FL incorporates rule based approach to solving a control problem rather than modeling them mathematically.

   E.G.:

   For the control system of an Industrial Furnace

   If the temperature is hot, then the pressure is rather high

   If the temperature is cold, then the pressure is very low

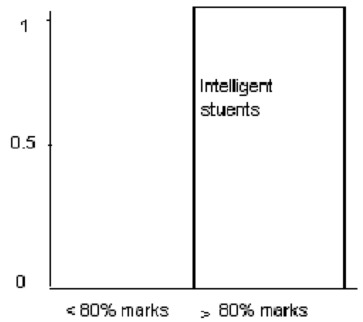
Fuzzy & Boolean Logic

1) In Boolean Logic every statement is true or false ie.. it has a truth value 1 or 0. Boolean sets impose rigid membership requirements.

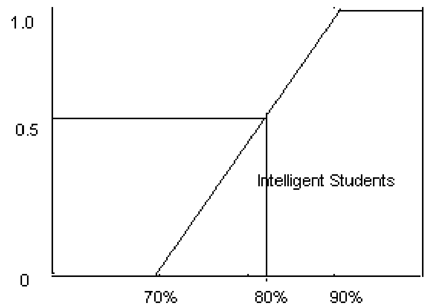
2) In contrast, fuzzy sets have more flexible membership requirements that allow for partial membership in a set.

3) Everything is a matter of degree and exact reasoning is viewed as a limiting case of approximate reasoning. Hence we can conclude that Boolean Logic is a subset of Fuzzy Logic.

Fuzzy & Boolean Logic



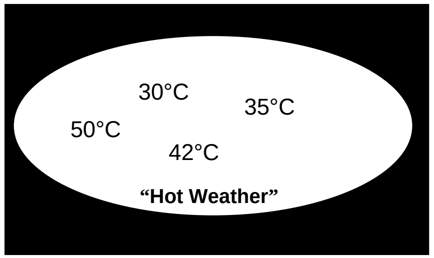
Traditional Sets



The fig indicates that a person securing 80% marks belongs to the set of Intelligent students with a degree of membership of 0.5

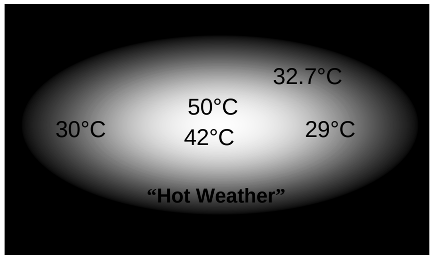
Fuzzy & Boolean Logic

Conventional (Boolean) Set Theory:



The boundary between hot and cold weather is fuzzy. Temperatures like 29°C and 30°C fall into that fuzzy region which is light grey.

Fuzzy Set Theory:



Weather can only be classified as warm or cold. 29°C and 30°C fall are included in cold weather although they fall in the category of warm weather.

Fuzzy sets

Definition : A fuzzy set A on a universe U is characterized by a membership function μ(x) that takes the values in the interval[0,1].

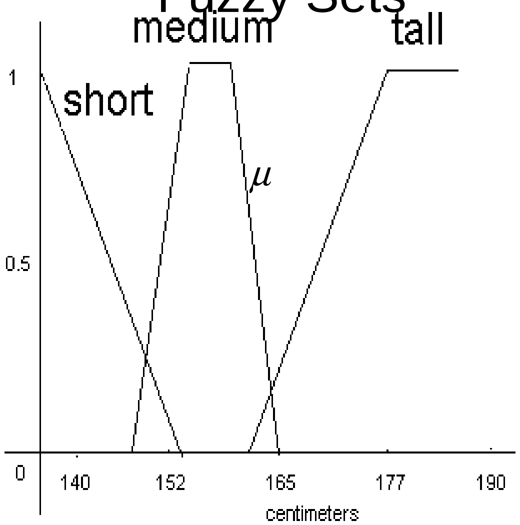
A fuzzy set A in U may be represented as a set of ordered pairs. Each pair consists of a generic element x and its grade of membership. It is given by

$$A=\{(x,\mu(x)) \mid x\in U\} \qquad \mu(x)=\{\mu_x^1,\mu_x^2,\ldots\ldots\mu_x^i\}$$

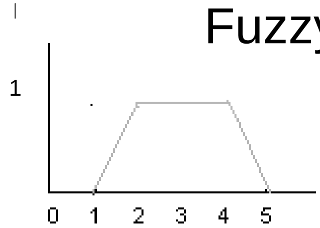
Term Sets : Term set contains fuzzy numbers. The term set for representing heights of people is T(X)= {Tall, Medium, Short}

$$T(x)=\{T_x^1,T_x^1,\ldots\ldots T_x^k\}$$

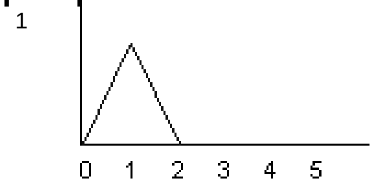
Fuzzy Sets



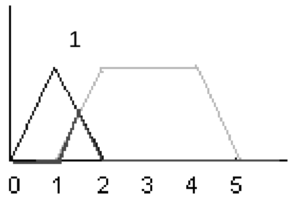
Fuzzy set properties



Let A be a fuzzy interval between 2 and 4

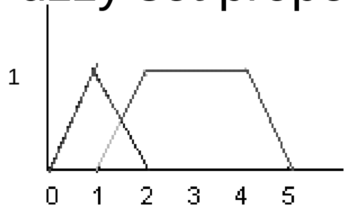


B be a fuzzy number about 1

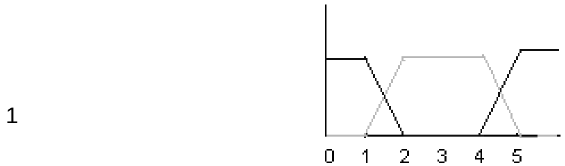


This figure shows the fuzzy set between 2 and 4 AND about 1

Fuzzy set properties…



The Fuzzy set *between* 2 *and* 4 **OR** *about* 1(again, it is the pink line).



The blue line is the **NEGATION** of the fuzzy set *A*

Fuzzy Sets - Properties...

Universe of boys - X = {Ram, Sham, Gopal, Anil}

Set of Tall - T = {0.9/Ram + 0.9/Sham + 0.6/Gopal}
μ (Ram) = 0.9; μ (Sham) = 0.9; μ (Gopal) = 0.6; μ (Anil) = 0

Set of Dark - D = {0.4/Ram + 0.8/Gopal + 0.7/Anil}
μ (Ram) = 0.4; μ (Sham) = 0; μ (Gopal) = 0.8; μ (Anil) =0.7

Tall ∪ Dark - T ∪ D = $\mu_T \vee \mu_D$ = max (μ_T, μ_D)
= {0.9/Ram + 0.9/Sham + 0.8/Gopal + 0.7/Anil}

Tall ∩ Dark - T ∩ D = $\mu_T \wedge \mu_D$ = min (μ_T, μ_D)
= {0.4/Ram + 0.6/Gopal}

Membership Function

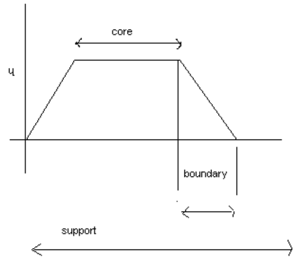
Membership function Maps elements of Fuzzy set to real numbered value in the interval 0 to 1

* $\mu_{\sim A}(x) \in [0,1]$

For Core elements μ(x) =1

For Support elements μ(x) > 0

For Boundary elements μ(x) > 0 but μ(x) ! =1



Based on membership functions the fuzzy sets are classified into two types

1) Normal Fuzzy set

A set is one whose membership function has at least one element whose degree of membership is equal to one.

Membership Function

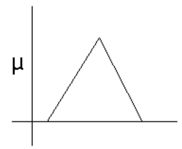
2) Convex Fuzzy set

Co A Convex Fuzzy set is described by the membership function whose membership values are strictly monotonically increasing or whose membership values are strictly monotonically decreasing or whose membership values are strictly monotonically increasing and then strictly monotonically decreasing for increasing values of elements in the universe.

Mathematical Expression:

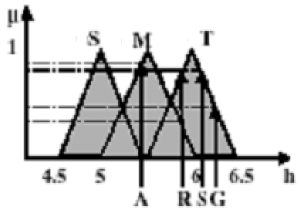
For any element x,y zin fuzzy set A,

the relation x<y<z implies



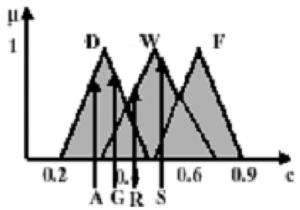
$$\mu_A(y) \le \min[\mu_A(x), \mu_A(z)]$$

Membership Function



Height Membership Function

Anil = 5.4	$\mu_M = 0.9$	$\mu_T = 0$
Ram = 5.11	$\mu_M = 0.5$	$\mu_T = 0.9$
Sham = 6.1	$\mu_M = 0$	$\mu_T = 0.9$
Gopal = 6.3	$\mu_M = 0$	$\mu_T = 0.6$

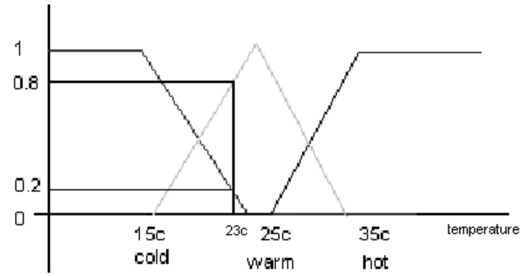


Complexion Membership Function

Anil = 0.27	$\mu_D = 0.7$	$\mu_W = 0$
Gopal = 0.32	$\mu_D = 0.8$	$\mu_W = 0.1$
Ram = 0.41	$\mu_D = 0.4$	$\mu_W = 0.6$
Sham = 0.52	$\mu_D = 0$	$\mu_W = 0.9$

Fuzzification

- Definition: The process of transforming crisp input values into Linguistic values is called Fuzzification.
- 1) Input values are translated into linguistic concepts, which are represented by fuzzy sets.
- 2) Membership functions are applied to the measurements and the degree of truth in each premise is determined.



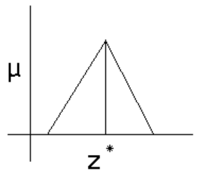
Defuzzification

It converts the fuzzy value into a "crisp" value. This process is often complex since the resulting fuzzy set might not translate directly into a crisp value. Physical systems need discrete values and hence Defuzzification is important.

The different methods of Defuzzification are

1) Max-Membership Principle: This method chooses the element with the maximum μ value.

$$\mu_c(z^*) \geq \mu_c(z)$$



Defuzzification

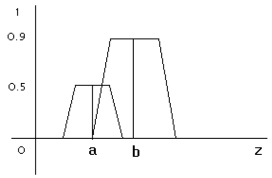
2) Centroid Method: The centroid defuzzification method finds the “balance” point of the solution fuzzy region by calculating the weighted mean of the output fuzzy region.

$$z^* = \frac{\int \mu_c(z).zdz}{\int \mu_c(z)dz}$$

3) Weighted Average Method: The weighted average method is formed by weighting each membership function in the output by its respective maximum membership value.

$$z^* = \frac{\sum \mu_c(\bar{z})\bar{z}}{\sum \mu_c(\bar{z})}$$

$$z^* = \frac{a(0.5) \ + \ b(0.9)}{0.5 \ + \ 0.9}$$



Linguistic Hedges

- Hedges are modifiers of fuzzy values and allow generation of fuzzy statements through mathematical calculations.
- Hedges act on fuzzy set’s membership function to modify it. Hedges play the same role in Fuzzy production rules that adjectives and adverbs play in English sentences.
- Depending on their impact on the membership function, the hedges are classified as concentrators, dilators and contrast hedges.

Linguistic Hedges

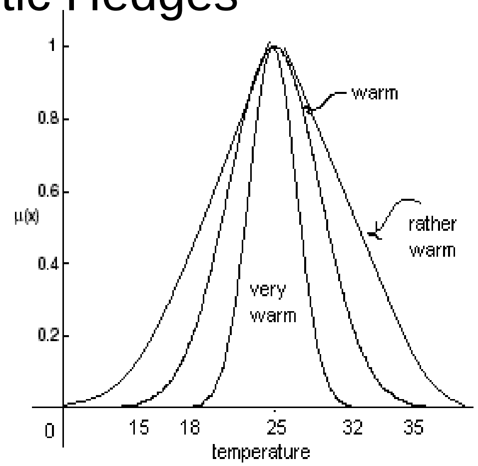
Concentrator hedge intensifies the fuzzy region.

$$\mu_{con(A)}(x) = \mu_A^n(x)$$

where n>=1.

Dilator hedge which dilutes the force of fuzzy set membership function. .

$$\mu_{dil(A)}(x) = \mu_A^{1/n}(x)$$



In the fig. “very warm” represents the concentrator

And “rather warm” represents dilator

Linguistic Hedges

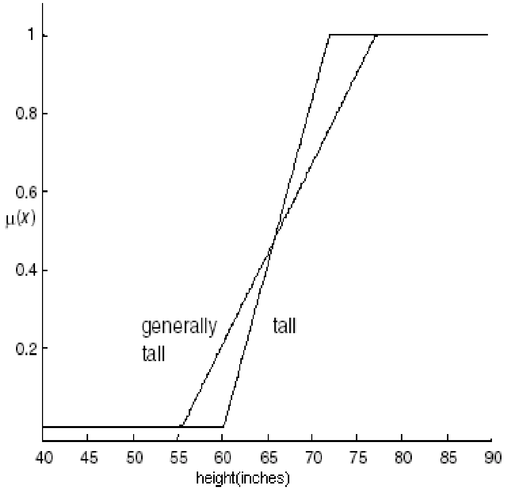
The **contrast hedges** change the nature of the fuzzy region by making it either less fuzzy(intensification) or more fuzzy(diffusion).

If μ is >= 0.5

$$\mu(A) = \frac{1}{2}(\mu_A^{\frac{1}{2}}(A))$$

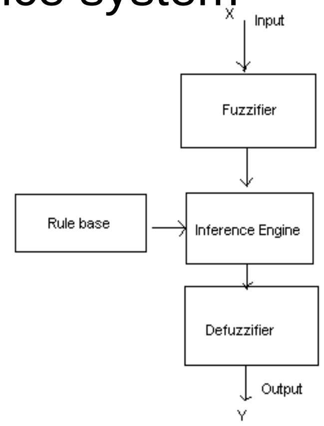
else if μ< 0.5

$$\mu(A) = 1 - \frac{1}{2}(\mu_A^{\frac{1}{2}}(A))$$



Fuzzy inference system

A fuzzy inference system essentially defines a nonlinear mapping of the input data vector into a scalar output using fuzzy rules.



Fuzzy inference system

Consider a multiinput and multioutput system. Let the input and output vectors be represented as follows:

$$x = (x_1, x_2, x_3, x_4, x_5)T$$

$$y = (y_1, y_2, y_3, y_4, y_5)T$$

The linguistic variable x in the universe of discourse is characterized by

$$T(x) = \{T_x^1, T_x^1, ... T_x^k\} T_x^i \qquad \mu(x) = \{\mu_x^1, \mu_x^2, ... \mu_x^i\}$$

Fuzzy inference system

. The inputs are

x1=years of education

x2= years of experience

The output is

y=salary.

Let T(x1) = {low,medium,high} for U in range [0-15].

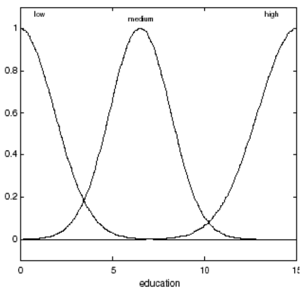
T(x2)= { low,medium,high} for U in range [0-30].

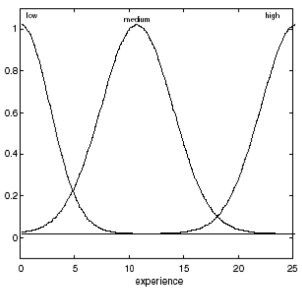
Let T(y) = {very low, low, medium, high, very high}

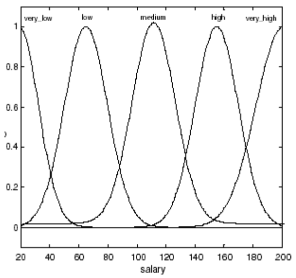
for U in range [2000, 20000].

Step 2 # Fuzzy inference system

Fuzzification of the input







Membership Function For X1

Membership function for X2

Membership Function for X3

Fuzzy inference system

A fuzzy rule base contains a set of fuzzy rules R. An example of the rule can be

"If education is high and experience is high, then the salary is very high"

In general for a multiinput and multioutput system, $R = (R_1, R_2, \ldots\ldots, R_N)$

The i[th] fuzzy rule is given by

$$R_i = If\ (x_1\ is\ T_{x_1}\ and\ \ldots.,\ x_p\ is\ T_{x_p})\ then$$
$$(y_1\ in\ T_{y_1}\ in,\ and\ \ldots\ldots\ y_q\ in\ T_{y_q})$$

The p precondition of R form a fuzzy set $(T_{x_1} \times T_{x_2} \times \ldots.. \times T_{x_p})$

And the consequent is given by the union of q independent outputs

$$(T_{y1} \cup T_{y2} \cup \ldots.. \cup T_{yp})$$

Fuzzy inference system

Interpreting an if-then rule is a three part process:

a) Resolve all fuzzy statements in the antecedent to a degree of membership between 0 and 1.

b) If there are multiple parts to the antecedent, apply fuzzy logic operators and resolve the antecedent to a single number between 0 and 1

c) Apply the implication method, using the degree of support for the entire rule to shape the output fuzzy set. For eg: consider an ith rule

$$R_i : if\ x_1\ is\ T_{x_1}\ and\ x_2\ is\ T_{x_2}\ then$$
$$Y\ is\ T_y^i$$

Fuzzy inference system

Then the **firing strength** or membership of the rule can be defined as

$$\alpha_i = \min(\mu_{x1}, \mu_{x2}, \ldots, \mu_{xn})$$
$$OR$$
$$\alpha_i = (\mu_{x1} * \mu_{x2} * \ldots.\mu_{xn})$$

Aggregation: Combining of two or more output fuzzy sets into a single composite output fuzzy set

$$\mu_y(w) = \max(\mu_y^1(w)\,\mu_y^2(w))$$

The result is the defuzzified to obtain a crisp value.

Fuzzy inference system

Eg: Consider the following values as input to the Fuzzy Inference System

Education(x1) = 15 years

Experience(x1) = 25 years

To find the output Salary(Y)

Fuzzification of Input: 15 years belongs to the fuzzy set high education with a membership of 1 and it doesn't belong to low education or medium education.

25 years belongs to fuzzy set high experience with a membership of 1 and doesn't belong to low experience or medium experience.

The firing strength of the rules that get fired is given by

If education is high and experience is low, then the salary is medium

min(1 , 0) = 0

If education is high and experience is medium, then the salary is high

min(1, 0) = 0

Fuzzy inference system

If education is high and experience is high, then the salary is very high.

min(1,1) = 1

If education is low and experience is high, then the salary is medium

min(0,1)=0

If education is medium and experience is high, then the salary is high

min(0, 1) = 0

The output is calculated as

max(0,0,1,0,0) = 1

Hence the salary is high. The degree of membership is 1 which corresponds to the crisp value of Rs 200,000.

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Fuzzy inference system

Advantage of Fuzzy Control Systems over conventional controllers.

The steps to be carried out in any Fuzzy Control System are :-

1) Identify the inputs and the outputs

2) Create a fuzzy membership function for each

3) Construct the Rule base

4) Decide how the action will be carried out.

Fuzzy inference system

Consider the design of a fuzzy controller for a steam turbine

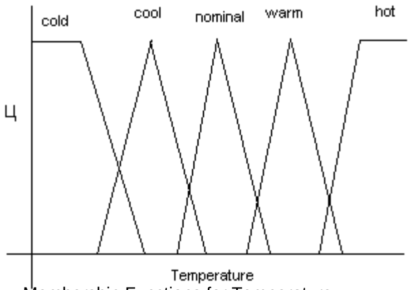
Step 1

1) The inputs are temperature and Pressure and the output is the state of Throttle (to decrease the flow of (steam or fuel to an engine) by a valve)

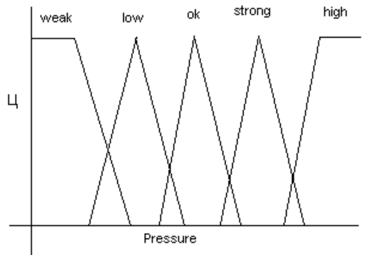
2) The term set for Temperautre is { cold, cool, nominal, warm, hot }

The term set for Pressure is { weak, low, ok, strong, high}

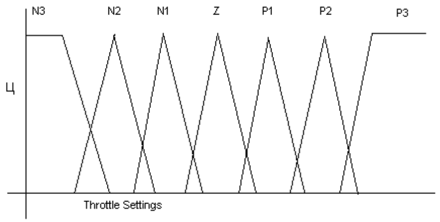
Fuzzy inference system



Membership Functions for Temperature



Membership Functions for Pressure



N3: Large negative.
N2: Medium negative.
N1: Small negative.
Z: Zero.
P1: Small positive.
P2: Medium positive.
P3: Large positive.

Fuzzy inference system

Step 3: Construct the Rule base.

rule 1: IF temperature IS cool AND pressure IS weak, THEN throttle is P3.

rule 2: IF temperature IS cool AND pressure IS low, THEN throttle is P2.

rule 3: IF temperature IS cool AND pressure IS ok, THEN throttle is Z.

rule 4: IF temperature IS cool AND pressure IS strong, THEN throttle is N2.

E.g.: Consider that the pressure is in low set with a membership of 0.7 and in ok state with a membership of 0.4. The temperature is assumed to be in cool state with membership of 1. The rules which fire are Rule2 and Rule3.

The output of Rule2 = 0.7*1 = 0.7

The output of Rule3 = 0.4*1 = 0.4

Fuzzy inference system

Step 4 : Deciding on the action: All the rules for which the antecedents have membership values greater than zero get fired. The Rule2 fires to a greater strength and hence it gets selected for deciding the output state. The value is defuzzified and the throttle is left as it is.

Advantages of Fuzzy Logic

- **Linguistic**, not numerical, variables are used, making it similar to the way humans think.
- **Simplicity** allows the solution of previously unsolved problems because they do away with complex analytical equations used to model traditional control systems
- **Rapid prototyping** is possible because a system designer doesn't have to know everything about the system before starting work.
- They're **cheaper** to make than conventional systems because they're easier to design.
- They have increased **robustness.**

Application Areas

Fuzzy Rule Based Systems

Fuzzy Nonlinear Simulations

Fuzzy Decision Making

Fuzzy Classification

Fuzzy Pattern Recognition

Fuzzy Control Systems